

Second-Best Mechanisms

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The VCG mechanism is the Only Efficient Mechanism

- Since the VCG mechanism is the only mechanism that
 - ▶ Makes truth-telling a dominant strategy
 - ▶ Implements the utilitarian rule
- And since the VCG mechanism yields a budget deficit,
- *There is no budget balanced, efficient mechanism for this social choice problem.*
- Ok then, the “first-best” is not attainable. What’s the best we can do with a budget-balanced mechanism? (The “second-best.”)

Budget Balanced Mechanisms

- From now on, we will consider mechanisms which never run a deficit.
- We will try to find the best among such mechanisms.
- i.e. closest to efficiency.

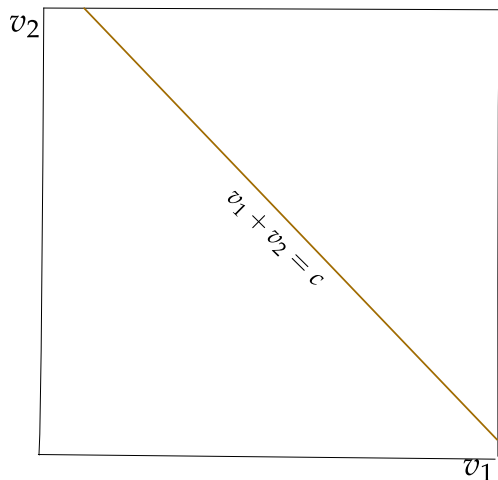
Espresso Machine Public Good Example

- Two individuals, with willingness to pay v_1 , v_2 for a public good.
- The cost to produce the public good is c .
- It is efficient to produce the good whenever $v_1 + v_2 \geq c$.
- But efficiency is not attainable without a budget deficit.
- As we look for second-best mechanisms, we now can choose both
 - ▶ The decision rule (in what cases to produce the good)
 - ▶ The transfer rule.

Mechanisms

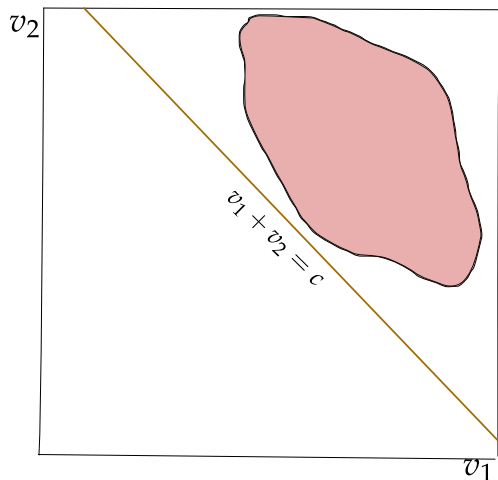
- Alternatives are on (the good is produced) and off (the good is not produced.)
- A mechanism is described by two functions
 - ▶ The *decision rule*, denoted $\alpha(\hat{v})$, which specifies the alternative.
 - ▶ The *transfer rule*, denoted $t(\hat{v})$, which specifies the transfer scheme.
- The only feasible mechanisms are those with $t_1(\hat{v}) + t_2(\hat{v}) \leq 50$ for all \hat{v} .
- We also want our mechanisms to be *incentive compatible*, that is we want truth-telling to be a dominant strategy.

Looking for Second-Best Mechanisms



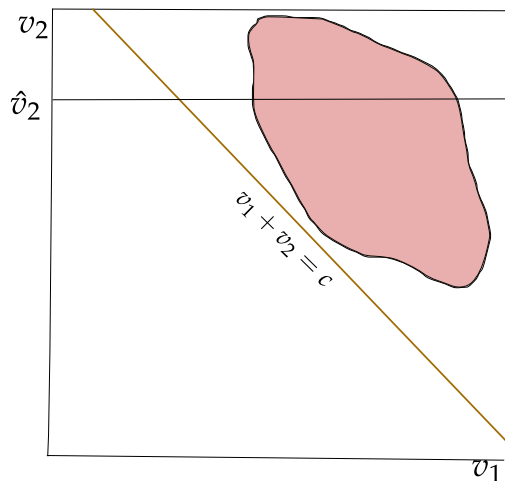
The public good problem.

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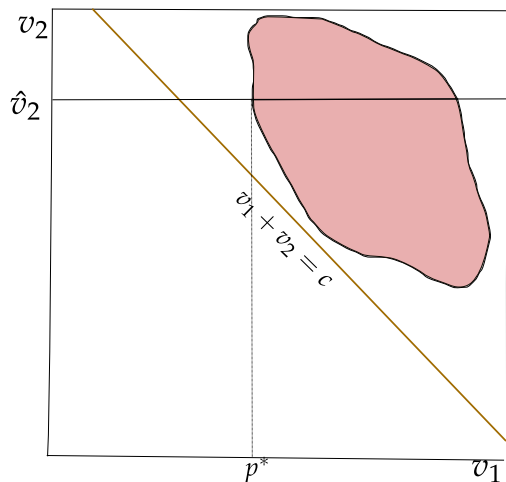
A picture of a decision rule. We want to see whether a transfer rule can be designed to make this an incentive-compatible mechanism.

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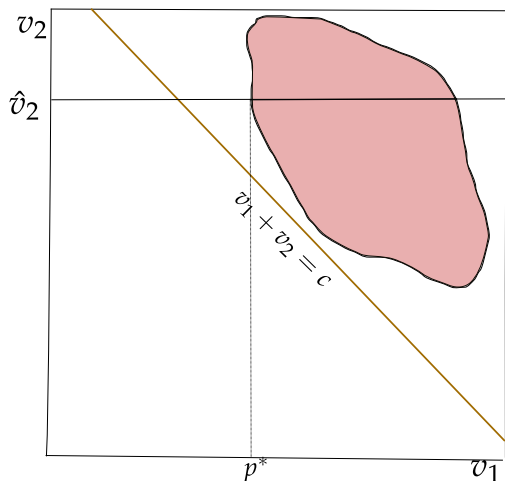
Suppose that 2 announces \hat{v}_2 .

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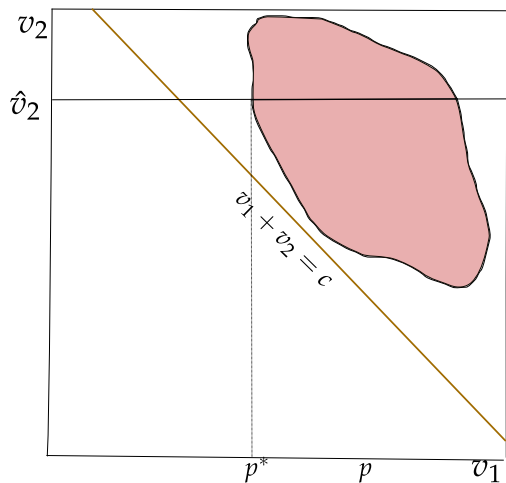
What should 1 be required to contribute if the good is produced? Note that the contribution cannot depend on 1's announcement \hat{v}_1 .

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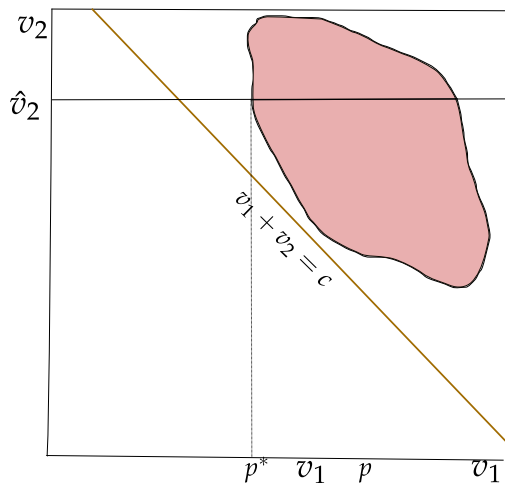
We will show incentive compatibility requires that the contribution should be p^* .

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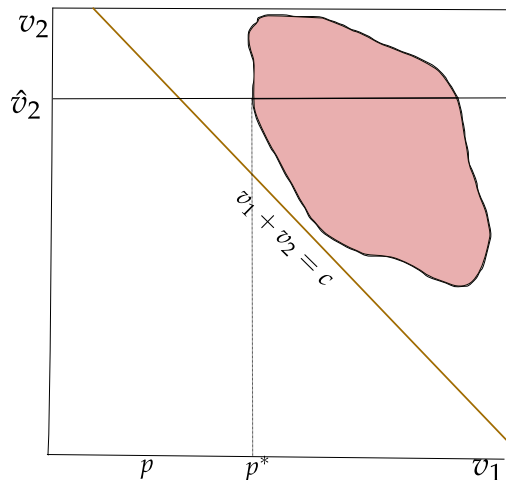
If instead it were higher than p^* , say at p , then...

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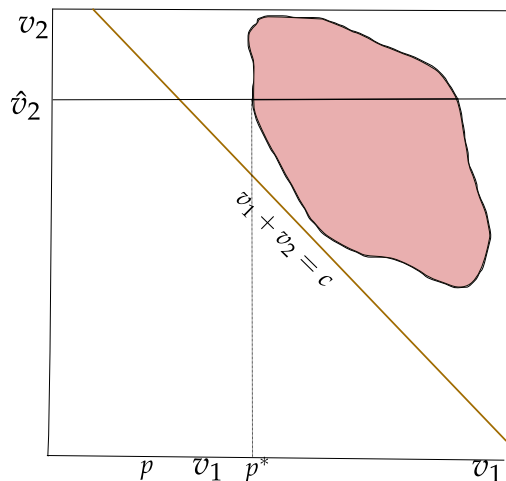
1 would have an incentive to understate his value when it is between p^* and p . (He would rather not have the good than to pay price $p > v_1$.)

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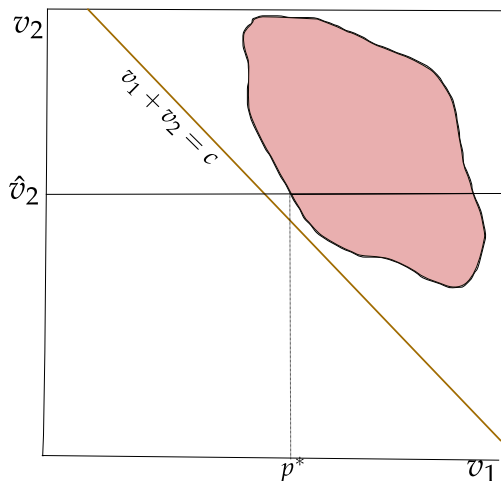
Likewise, the contribution cannot be below p^* ...

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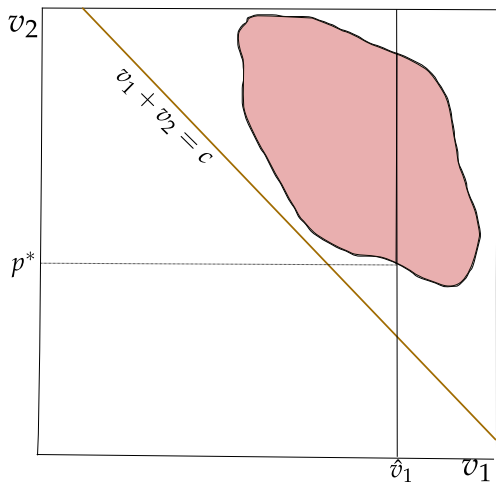
Because then 1 would have an incentive to overstate his value when it is between p and p^* . (He would rather have the good and pay only $p < v_1$.)

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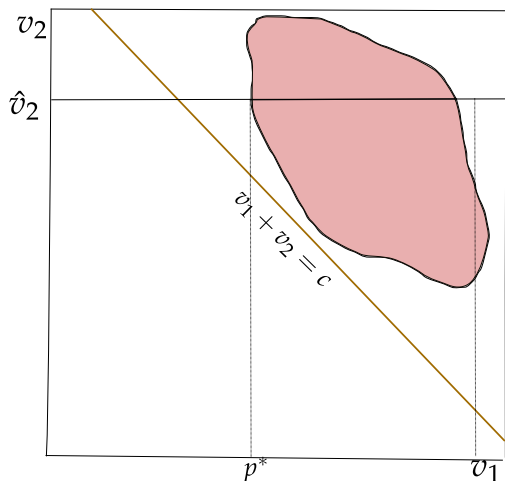
We use this same procedure to determine 1's contribution as a function of \hat{v}_2 .

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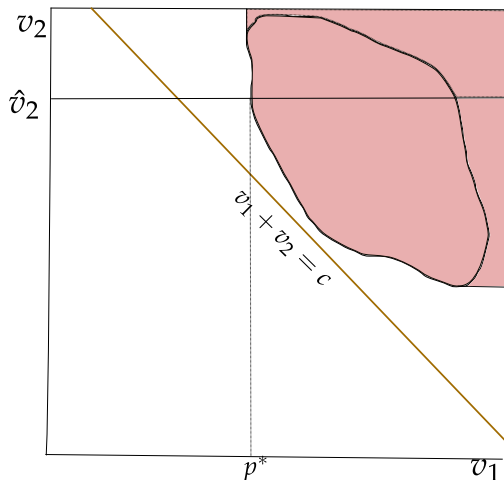
And for player 2's contribution as a function of \hat{v}_1 .

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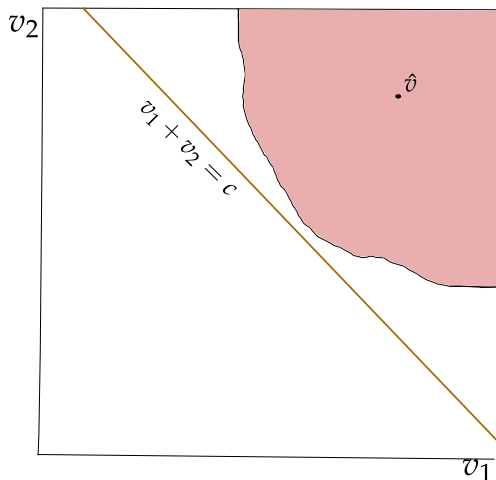
Going back to player 1, what happens when his value is to the right of the cloud?

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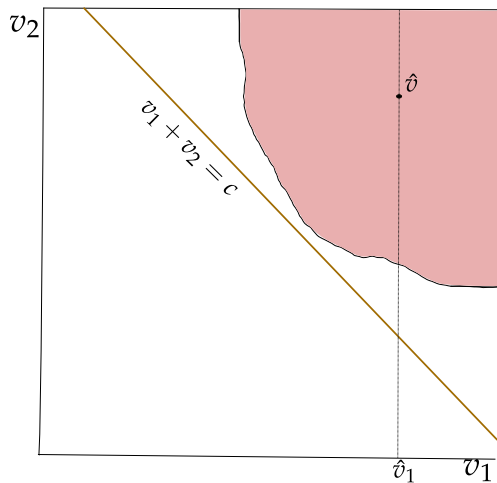
The set in which the good is produced must extend to the northeast.

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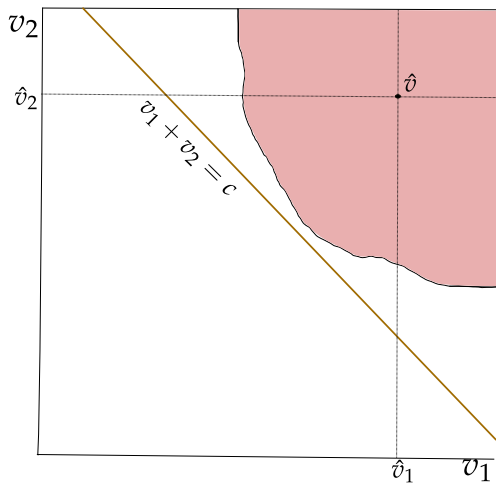
Now we will figure out if there is a deficit. Suppose they announce \hat{v} in the region where the good is produced.

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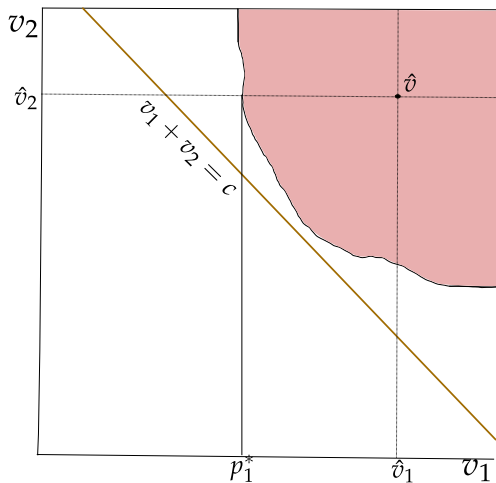
So 1 has announced \hat{v}_1

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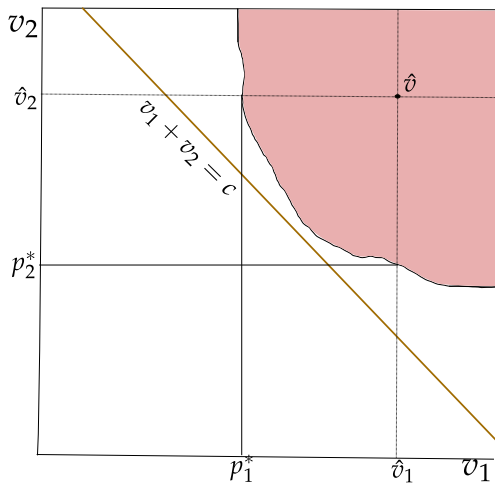
And 2 has announced \hat{v}_2 .

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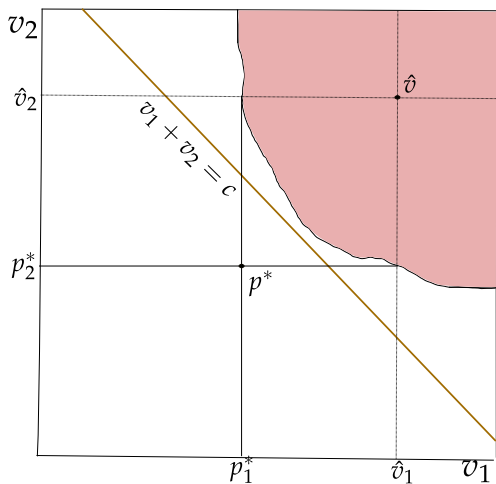
We can figure out what 1 should contribute,

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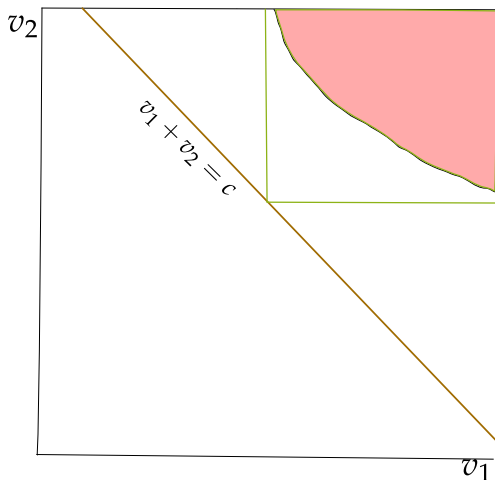
And what 2 should contribute.

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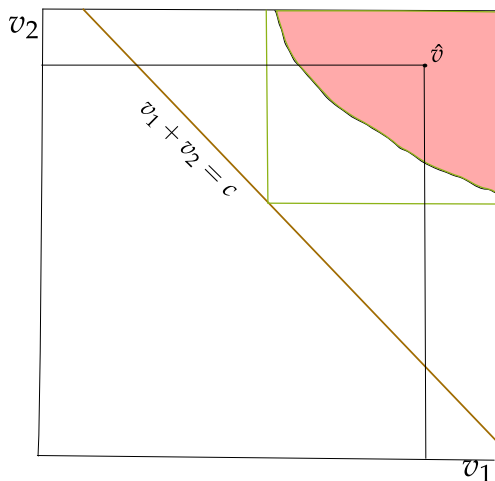
The point p^* indicates the pair of payments (p_1^*, p_2^*) . It is below the diagonal line. That means $p_1^* + p_2^* < c$. There is a budget deficit.

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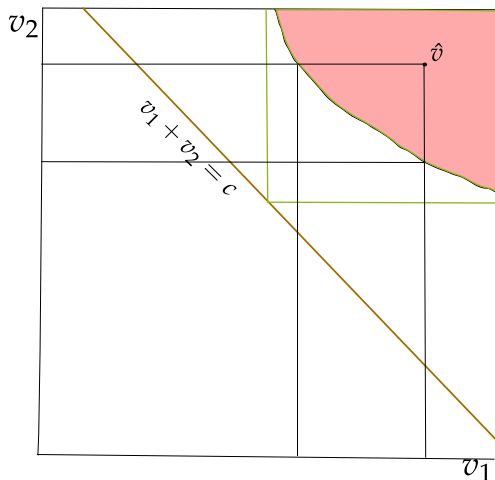
This happened because the set is not contained in any rectangle whose lower-left corner is on the diagonal line.

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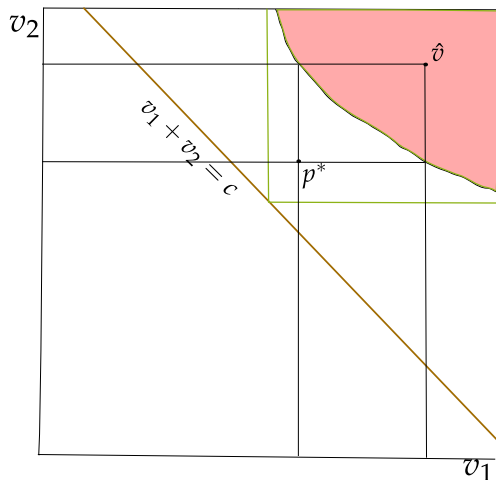
On the other hand, if we pick a set which is contained inside such a rectangle, there is guaranteed not to produce a deficit, regardless of what the players announce.

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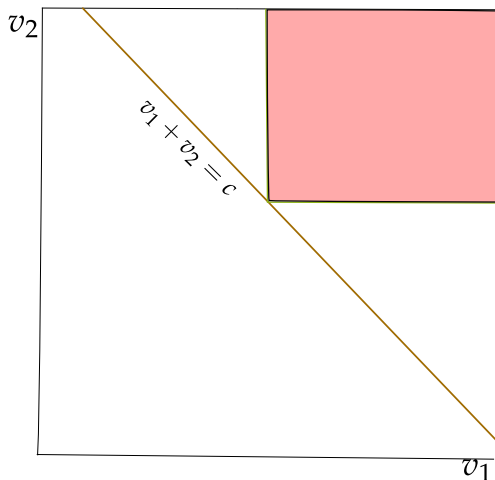
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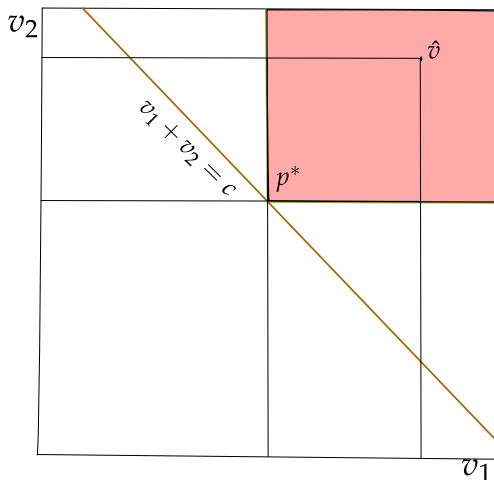
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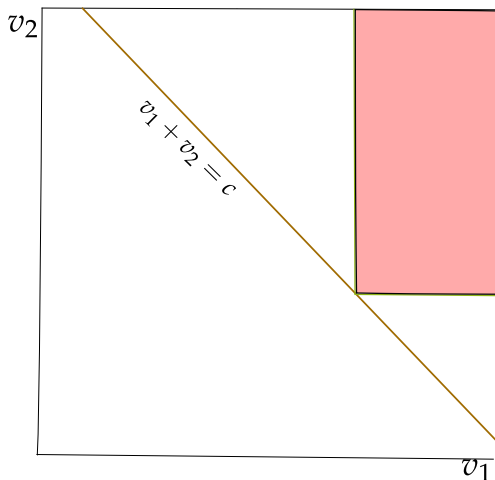
So, since we know that the set must be included inside a rectangle, we might as well make it the whole rectangle.

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Such a mechanism is budget balanced (exactly.)

Looking for Second-Best Mechanisms



Any rectangle will do. Different rectangles correspond to different ways of dividing the cost.

Summary

- Any incentive compatible mechanism which does not run a deficit must be contained in a rectangle.
- The most efficient of these use up an entire rectangle.
- These are the “second best” mechanisms: the best among those that never run a deficit.
- Any rectangle will is a second-best mechanism.
- The choice of rectangle determines how the cost of the public good will be split.
- Thus, a second-best mechanism reduces to this:
 - ▶ *First*, we agree on how the cost would be split.
 - ▶ Then the players say yes or no.
 - ▶ If one says no, then the good is not produced.
 - ▶ If both say yes, then the good is produced and the cost split according to the pre-specified rule.
- This is just the split-the-cost mechanism (for potentially unequal cost shares.)
- We have shown that it is impossible to improve upon that simple mechanism.