Welfare Economics

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Social Choices

We will study situations like the following

- Some public policy decision (the "alternatives")
- affects all members of some group (the "society") and
- individuals have (typically opposing) policy preferences.

Examples Abound

Sounds abstract, but its part of everyday life. Examples (items in blue are web links to related stories):

- Ad placement on Google
- NIN ticket sales
- "Distressed" assets.
- Public School Choice.
- Sustainable Fisheries.
- Kidney Exchange.
- Designer Dress Dibs.
- Twitter!!

Formal Definitions

A Social Choice Problem consists of

- A set of individuals i
- A set $\mathcal{A}$ of alternatives A.
- For each individual i, a preference ordering $\succ_i$ which ranks alternatives.

So $A \succ_i B$ means that i prefers A over B. (Pairwise rankings, transitivity.)

Social Welfare Functions

What we are after is a *social ranking* $\succ^*$.
The social ranking

1. Is our principle for deciding which outcomes are "good for society."
2. Naturally should depend on the preferences of the individuals.
3. This dependence is described abstractly by a *Social Welfare Function.*

A Social Welfare Function

Definition

A social welfare function is a mathematical function which takes as an input the list of preferences $(\succ_1, \succ_2, \ldots, \succ_n)$ and produces as an output a single preference ranking $\succ^*$.

Examples:

1. (2 alternatives, odd number of individuals) Majority rule.
2. Plurality Rule (a common voting system.) Details
3. Borda criterion (rank-order, or "point-system" voting.) Details
4. Rawlsian SWF (maximize the welfare of the worst-off individual.) Details

Minimal Requirements for a SWF

Definition

A SWF satisfies Universal Domain (UD) if every possible preference list input results in a well-defined social ranking output.

To illustrate, consider an example of a SWF which violates UD: pairwise majority rule.

Individual 1) $A \succ B \succ C$

Individual 2) $B \succ C \succ A$

Individual 3) $C \succ B \succ A$

Minimal Requirements for a SWF

Definition

A SWF satisfies Pareto if it respects unanimity. In formal terms, Pareto is satisfied if whenever it happens that for some pair of alternatives A, B, every individual i ranks $A \succ_i B$ the social ranking that is output also ranks $A \succ^* B$.

Independence of Irrelevant Alternatives

The last requirement is a bit more subtle. It is best to introduce it by means of an example. Recall the plurality rule.

Suppose there are 11 individuals and 3 alternatives labeled B, G, and N. The individuals' preferences are as follows.

Group 1) $B \succ G \succ N$ (4 individuals)

Group 2) $G \succ N \succ B$ (3 individuals)

Group 3) $G \succ B \succ N$ (2 individuals)

Applying the plurality rule results in $G \succ^* B \succ^* N$. (In fact a majority prefer G to B.)

Now consider what happens if the preferences are slightly different.

Group 1) $B \succ G \succ N$ (4 individuals)

Group 2) $G \succ N \succ B$ (3 individuals)

Group 3) $N \succ G \succ B$ (2 individuals)

Only the preferences of the last group are different. No individual has changed their ranking of B versus G. Yet now the plurality rule produces a reversal: $B \succ^* G \succ^* N$.

In the preceding example, the social ranking of G versus B was reversed despite no change in any individual's ranking of G versus B. The next condition asks that a SWF never suffer from this defect.

Definition

A SWF satisfies Independence of Irrelevant Alternatives if the social ranking of A versus B depends only on the individuals ranking of those two alternatives. In formal terms, if we are given two lists of preferences which are identical in terms of each individual's ranking of A and B, then the SWF should output the same social ranking of A versus B.

A Good SWF?

- We have argued that a good SWF should satisfy UD, Pareto, and IIA.
- But these are *minimal* requirements of a SWF.
- Note that they say very little about equity, fairness, etc.
- So a SWF should satisfy *at least* these, plus probably more.

To illustrate, here is a SWF which satisfies the three conditions but which is clearly *not* a good SWF.

Dictatorship

A SWF is a *dictatorship* if for some individual i, the social ranking $\succ^*$ is always exactly the same as $\succ_i$ regardless of the preferences of individuals other than i.

In this case, we say that i is the dictator. A dictatorship satisfies

1. UD because it always gives an answer ($\succ^* = \succ_i$)
2. Pareto because
    1. If it is unanimous that A is better than B then
    2. In particular $A \succ_i B$ for the dictator i and therefore
    3. $A \succ^* B$ as required by Pareto.
3. IIA because the ranking of any A and B depend only on the dictator's ranking of A and B.

Can we add to our conditions?

We can try to think of other SWF that satisfy UD, Pareto and IIA.

1. Majority rule: not defined for more than 2 alternatives (therefore fails UD)
2. Plurality rule: fails IIA.
3. Pairwise Majority rule fails UD.
4. Borda criterion? (exercise)

Arrow's Impossibility Theorem

The only SWF that can satisfy UD, Pareto, and IIA is dictatorship.
For an idea of how the proof works, see Wikipedia

Plurality Rule

- Determine which alternative has the most "top choice votes."
- That alternative is placed at the top of the social ranking.
- Now remove that alternative from the individual rankings (leaving the rest intact.)
- Determine which alternative (among those that remain) now has the most "top choice votes."
- That alternative is places second in the social ranking.
- Continue until all alternatives are ranked.

Borda Criterion

Suppose there are n alternatives.

- Begin with individual i. Assign points to alterantives as follows.
  - i's most-preferred alternative receives n points.
  - i's second most-preferred alterantive receives $n-1$ points.
  - Continue in this way, ultimately assigning 1 point to i's least-preferred alternative.
- Now do this for all individuals and add up the points each alternative receives.
- Rank the alternatives according to their total points.

Rawlsian SWF

Suppose there are n alternatives

- Start with an alternative A.
- Find the individual i who ranks A the lowest.
- Assign a score to A equal to the position in i's ranking (a number between 1 and n.)
- Do the same for all alternatives. (For each alternative there will usually be a different i who ranks it the lowest.)
- Finally, rank the alternatives in *decreasing* order according to this score.

Rawlsian SWF: Example

Individual 1) $A \succ B \succ C$

Individual 2) $A \succ B \succ C$

Individual 3) $C \succ B \succ A$

The Rawlsian SWF gives a score of 3 to both C and A because for each of them there is at least one individual who ranks it third. B gets a score of 2 because the lowest it appears on any list is second. Thus B is ranked at the top of the social ranking since it has the lowest score. We use whatever tie-breaking rule we want (as long as it is pre-determined) to rank A and C after B.